

Physics-Informed Machine Learning for Monitoring Natural Hazards in Digital Twins

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Outline

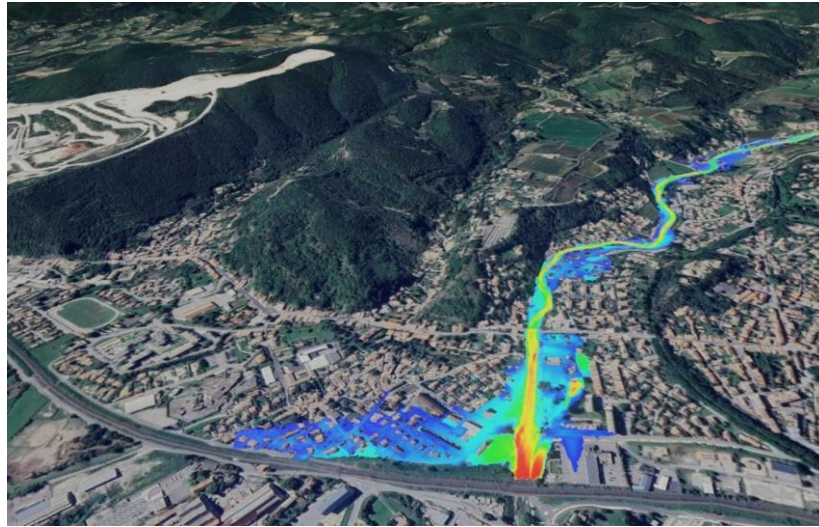
- ✓ What is a **Digital Twin**? The DITRISC project
- ✓ **Scientific challenges in modelling and simulation**, model classification
- ✓ **Some examples of natural hazards**, PDE framework
- ✓ **Synergy between models and data**: data assimilation
- ✓ **Data for natural hazard** monitoring
- ✓ A short introduction to **Deep Learning**, from data-based learning to Physics-Informed learning
- ✓ **PINN approach**
- ✓ **A case study**: A DT for flood monitoring
- ✓ **Conclusions**

Digital Twins for environmental applications

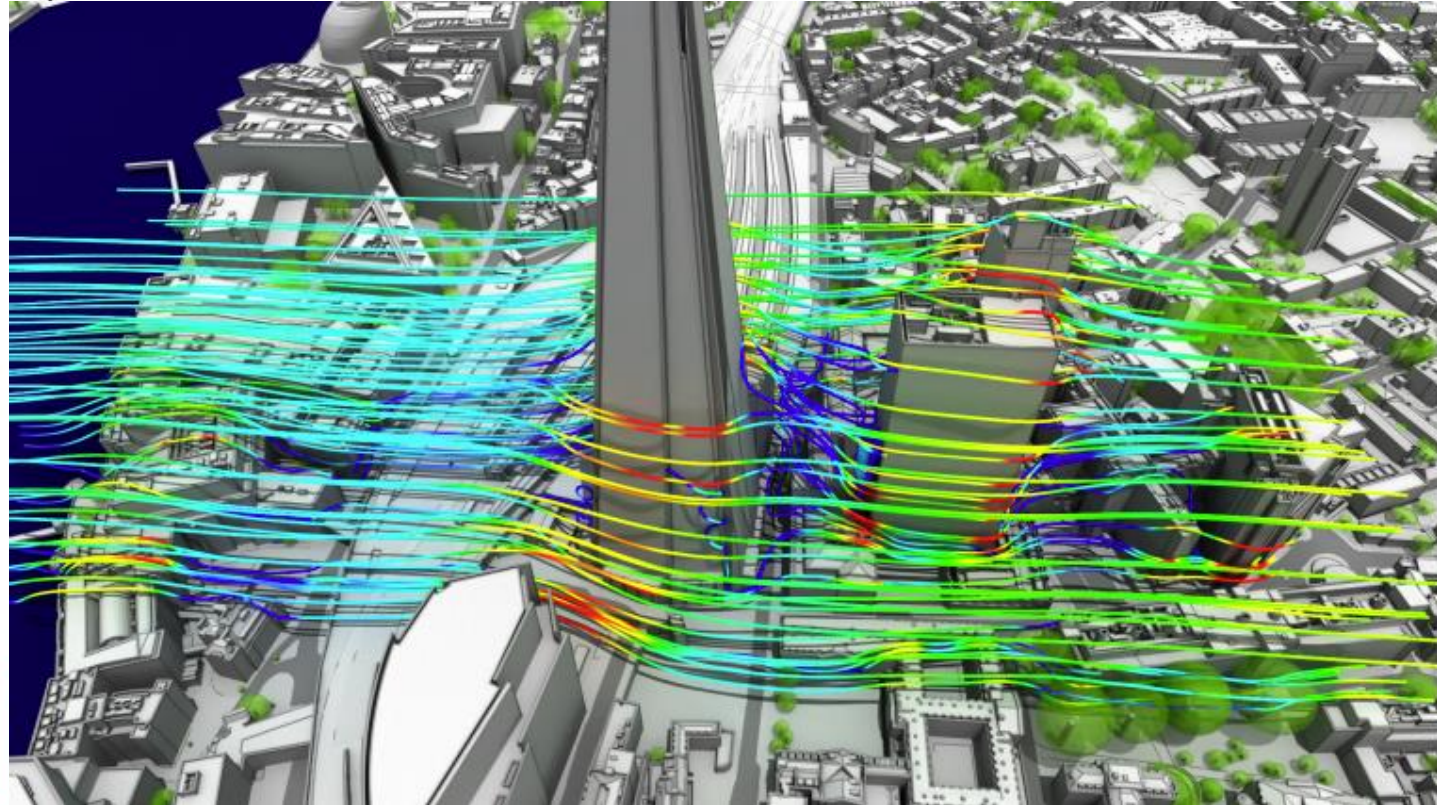
MODELS

- ▶ "A digital twin is a virtual representation synchronized with physical things (data), people or processes" (Digital shadows)

<https://www.gim-international.com/content/article/can-digital-twin-techniques-serve-city-needs>



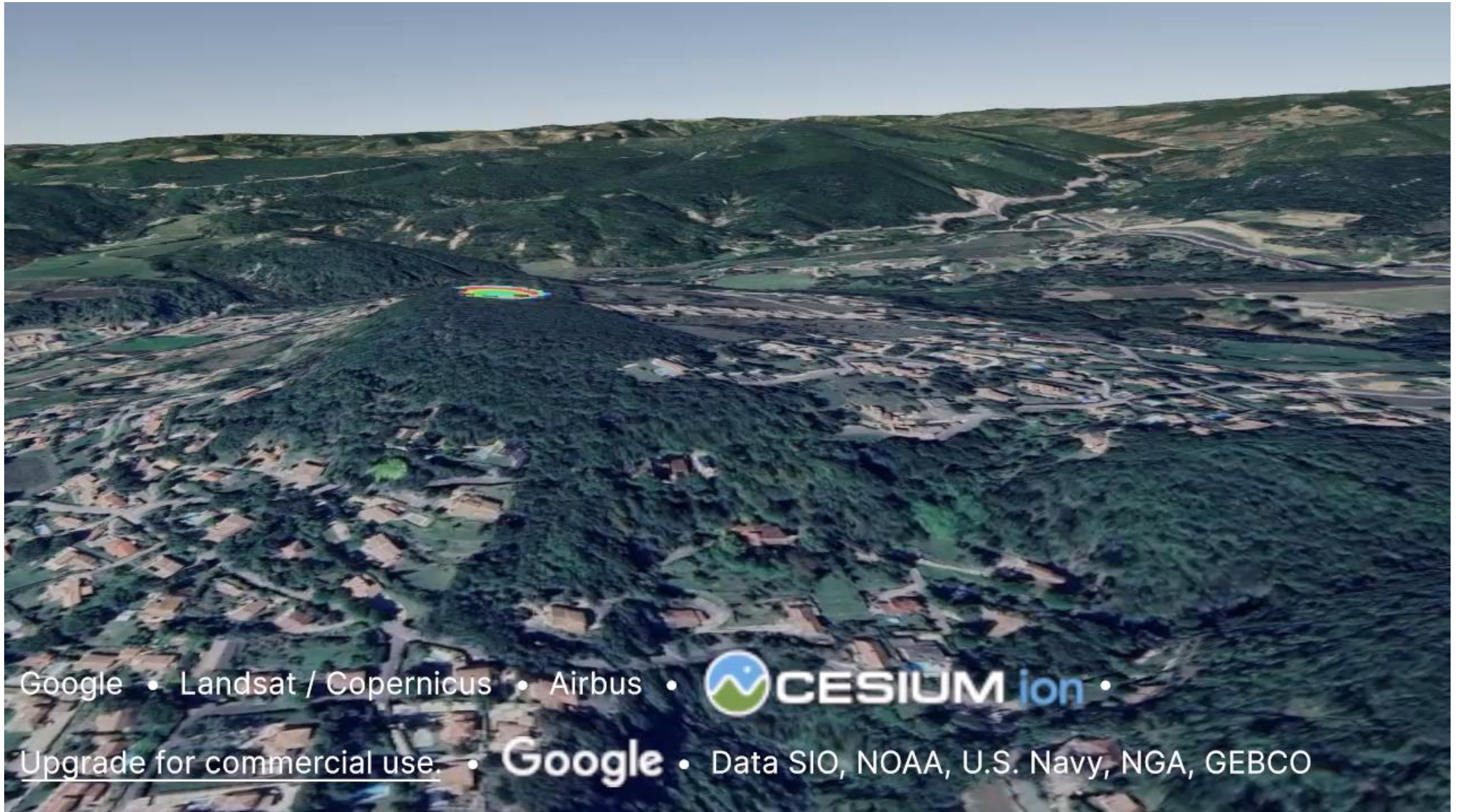
Scenario of a flood in the Frayol watershed,
Le Teil city, Ardèche, France
with Virtual Reality - P. Bellemain, Gipsa-lab 2024
DITRISC project



- ▶ For monitoring and diagnosis (early detection), prediction : scenarios elaboration, decision-making support

Digital Twin for environmental applications

P. Bellemain, GIPSA-lab, 2025

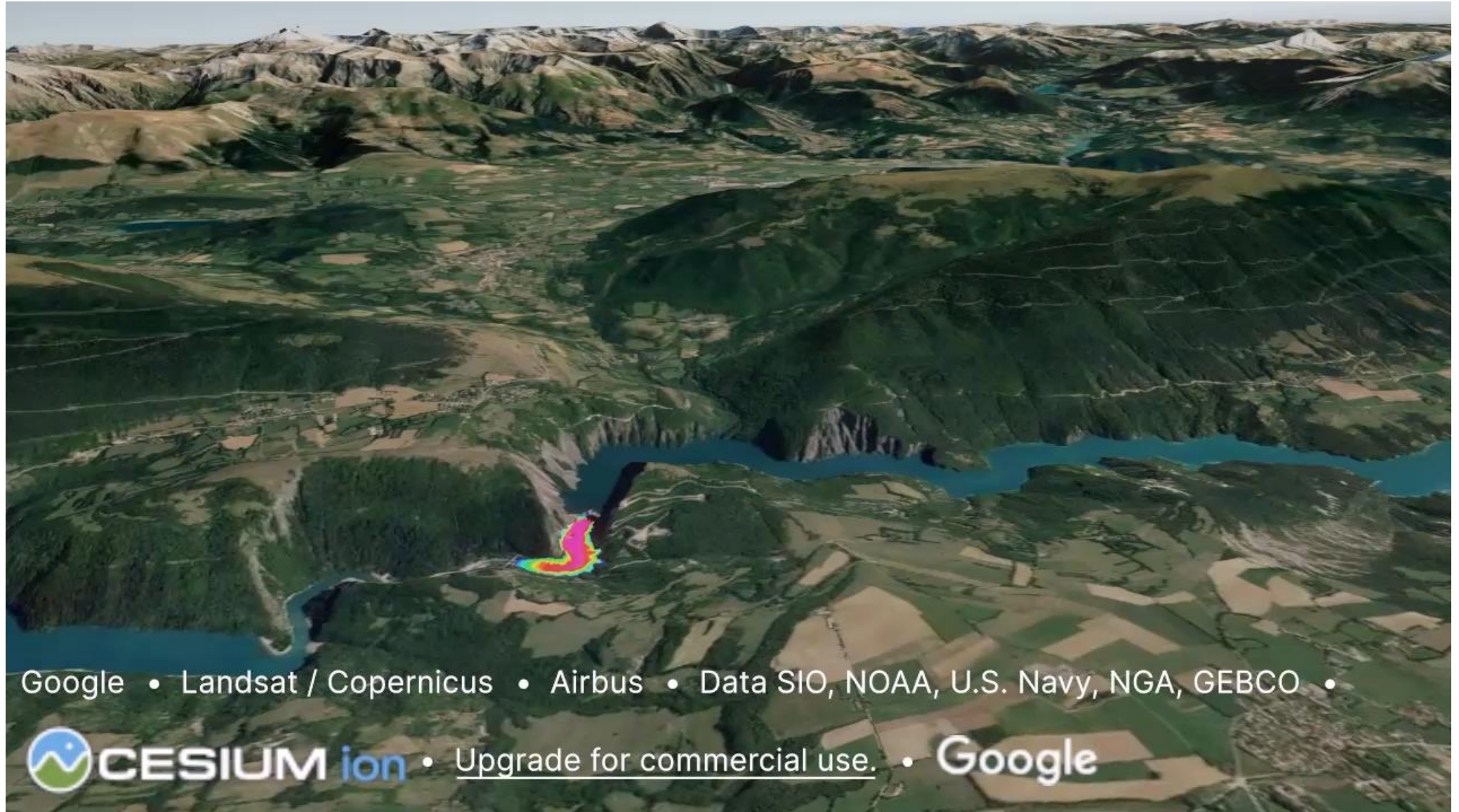
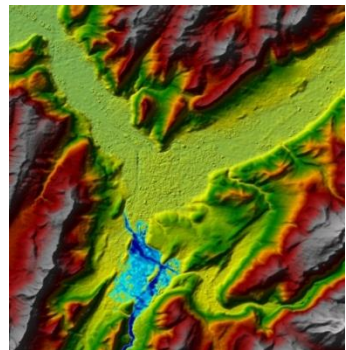


Digital Twin for environmental applications

P. Bellemain, GIPSA-lab, 2025

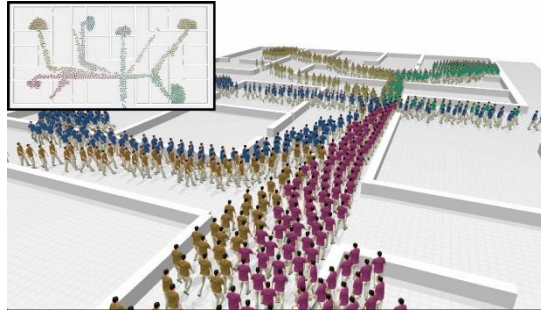


Monteynard dam

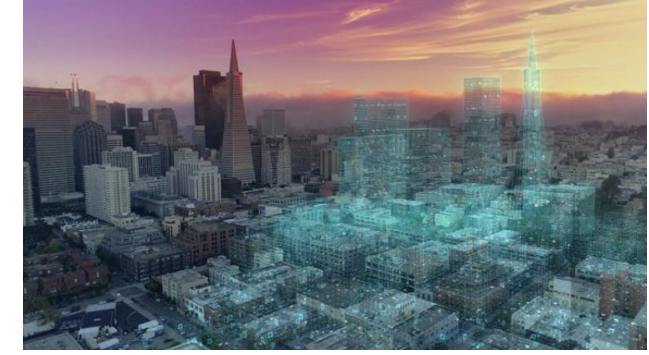


Scientific challenges of modelling and simulation for DT

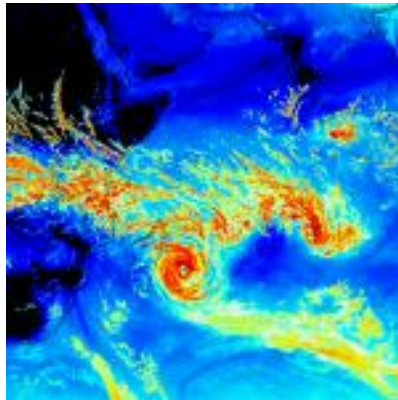
- ✓ Understanding cascading or coupled dynamics
- ✓ Managing different spatial and temporal scales
- ✓ Including human behavior



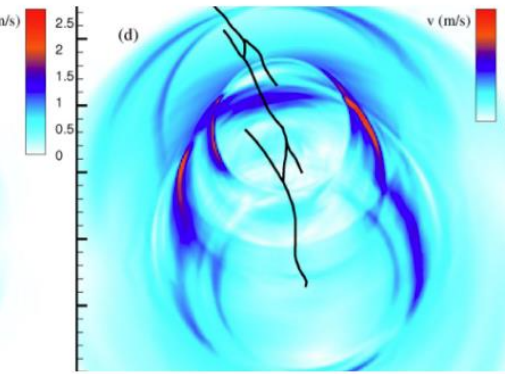
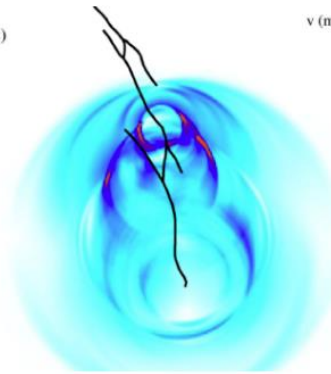
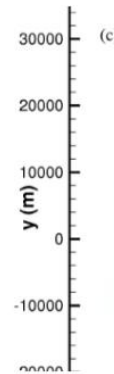
Crowd evacuation simulation



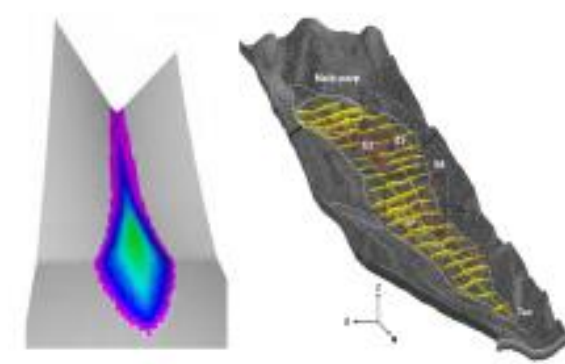
<https://aecmag.com/news/digital-twins-for-a-sustainable-built-environment/>



Macroscale
Cyclone simulation



Mesoscale
Earthquake simulation



Microscale
Landslide simulation



<https://theconversation.com/modeliser-le-climat-grace-au-calcul-scientifique-70461>

Model classification: Knowledge vs data-based models

Von Neumann paradigm

Physical modelling

- ✓ Compliance with the laws of Physics
 - ✓ Interpretability
 - ✓ Parametrization
-
- ✓ Theoretical investment
 - ✓ Calibration may be complex
 - ✓ Complex phenomena that are sometimes difficult to take into account
 - ✓ Simulation cost for large-scale models

Shallow Water Equations on a sphere

$$\frac{\partial \mathbf{u}_c}{\partial t} = -P \begin{bmatrix} (\mathbf{u}_c \cdot P \nabla_c) u_c + f(\mathbf{x} \times \mathbf{u}_c) \cdot \hat{\mathbf{i}} + g(P \hat{\mathbf{i}} \cdot \nabla_c) h \\ (\mathbf{u}_c \cdot P \nabla_c) v_c + f(\mathbf{x} \times \mathbf{u}_c) \cdot \hat{\mathbf{j}} + g(P \hat{\mathbf{j}} \cdot \nabla_c) h \\ (\mathbf{u}_c \cdot P \nabla_c) w_c + f(\mathbf{x} \times \mathbf{u}_c) \cdot \hat{\mathbf{k}} + g(P \hat{\mathbf{k}} \cdot \nabla_c) h \end{bmatrix}$$

$$\frac{\partial h^*}{\partial t} = -(P \nabla_c) \cdot (h^* \mathbf{u}_c)$$

AI – Machine Learning

Data-based modelling

- ✓ Easier access to modeling
 - ✓ Continuous learning and improvement
 - ✓ Low simulation cost once learned
-
- ✓ Weak interpretability
 - ✓ Compliance with the laws of Physics not guaranteed
 - ✓ High sensitivity to data quality

$$y(t) = \Phi_{RN}(y(t-1), y(t-2), \dots, y(t-n), u(t-1), u(t-2), \dots, u(t-m))$$

Auto-regressive surrogate model

Modelling of natural hazards – some examples

Floods, tsunami, landslides, avalanches dynamics

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(Hu) + \frac{\partial}{\partial y}(Hv) = 0, \quad H = h - b$$

Shallow water equations
(A. Barré de Saint Venant, 1871)

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + g \frac{\partial h}{\partial x} = -\tau_x$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + g \frac{\partial h}{\partial y} = -\tau_y$$

h = absolute height, u , v = velocity in x , y directions



Google images



Modelling of natural hazards – some examples

Advection-Diffusion-Reaction – Wildfire dynamics

$$\begin{aligned}\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} &= k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Sr(T) + \lambda(T - T_a), \\ \frac{\partial S}{\partial t} &= -\beta Sr(T), \\ r(T) &= e^{-\alpha/(T-T_a)}, T > T_a, r(T) = 0, T \leq T_a\end{aligned}$$

Arrhenius law of combustion

$T(x, y, t)$ = distributed temperature in the ground layer, T_a = ambient temperature, $S(x, y, t)$ = distributed mass fraction of fuel, (v_x, v_y) = air velocity, k = diffusion coefficient



Google images

Modelling of natural hazards – some examples

Elastic Wave equations – Seismic dynamics

$$\begin{aligned}\rho \partial_{t^2}^2 u(t, x) &= \nabla \cdot \sigma(t, x) + f(x) \\ \sigma(t, x) &= \lambda \nabla \cdot u(t, x) I + \mu [\nabla u(t, x) + \nabla u(t, x)^T]\end{aligned}$$

where $u(t, x)$ = displacement vector field and $\sigma(t, x)$ = stress tensor; $\rho(x)$ = density of the elastic medium, and $\lambda(x)$ and $\mu(x)$ are Lamé elastic constants; $f(x)$ = seismic source distribution



Séisme du Teil - 11 novembre 2019

BRGM

Modelling of crowd dynamics in evacuation situations – Hughes' model

PhD thesis – COCHAIR
- PEPR IRiMa

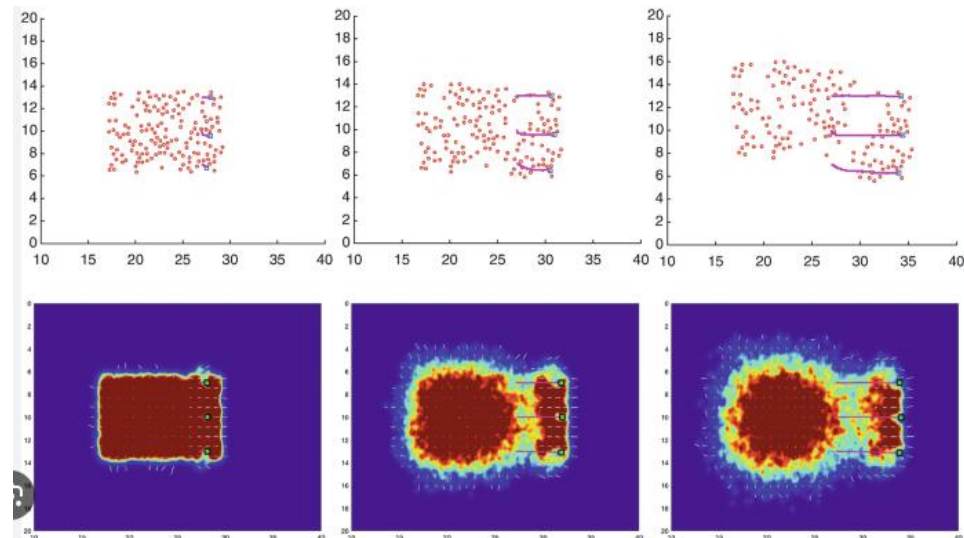
 **Density of individuals** as a function of time t and space coordinates x

$$\partial_t \rho - \nabla \cdot (\rho f(\rho)^2 \nabla \phi) = 0 \quad \text{in } Q_T, \quad Q_T := (0, T) \times \Omega$$

$|\nabla \phi| = \frac{1}{f(\rho)}$ in Q_T .  Pedestrians seek to minimize their (accurately) estimated travel time, but modify their velocity to avoid high densities. The potential is thus a solution of the **Eikonal equation**

$v = f(\rho) u$, $|u| = 1$, where $f(\rho)$ is a non-increasing function that attains the value one at $\rho = 0$, is positive for $0 < \rho < 1$ and zero at $\rho = 1$.

$$u = -\frac{\nabla \phi}{|\nabla \phi|}.$$



Multi-agent approach

Macroscopic approach

General framework for spatio-temporal modelling: **PDEs**

$$\mathcal{L}(u(x, t), \theta) = g$$

$$\mathcal{B}(u(x, t), \theta) = f$$

$u(x, t)$ is a physical quantity, function of space x et time t .

θ is a vector of parameters

$\mathcal{L}$ and $\mathcal{B}$ are the differential and boundary/initial condition operators, respectively.

$\mathcal{L}$ is a function of partial derivatives $\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \dots, \frac{\partial^k}{\partial x^k}$.

DT: Increasing synergy between models and data

For better reality representation and forecasting capabilities

Findings

Complex multi-scale spatio-temporal
Phenomena

Unstationarity & uncertainty

Massive and heterogeneous data,
weak signals

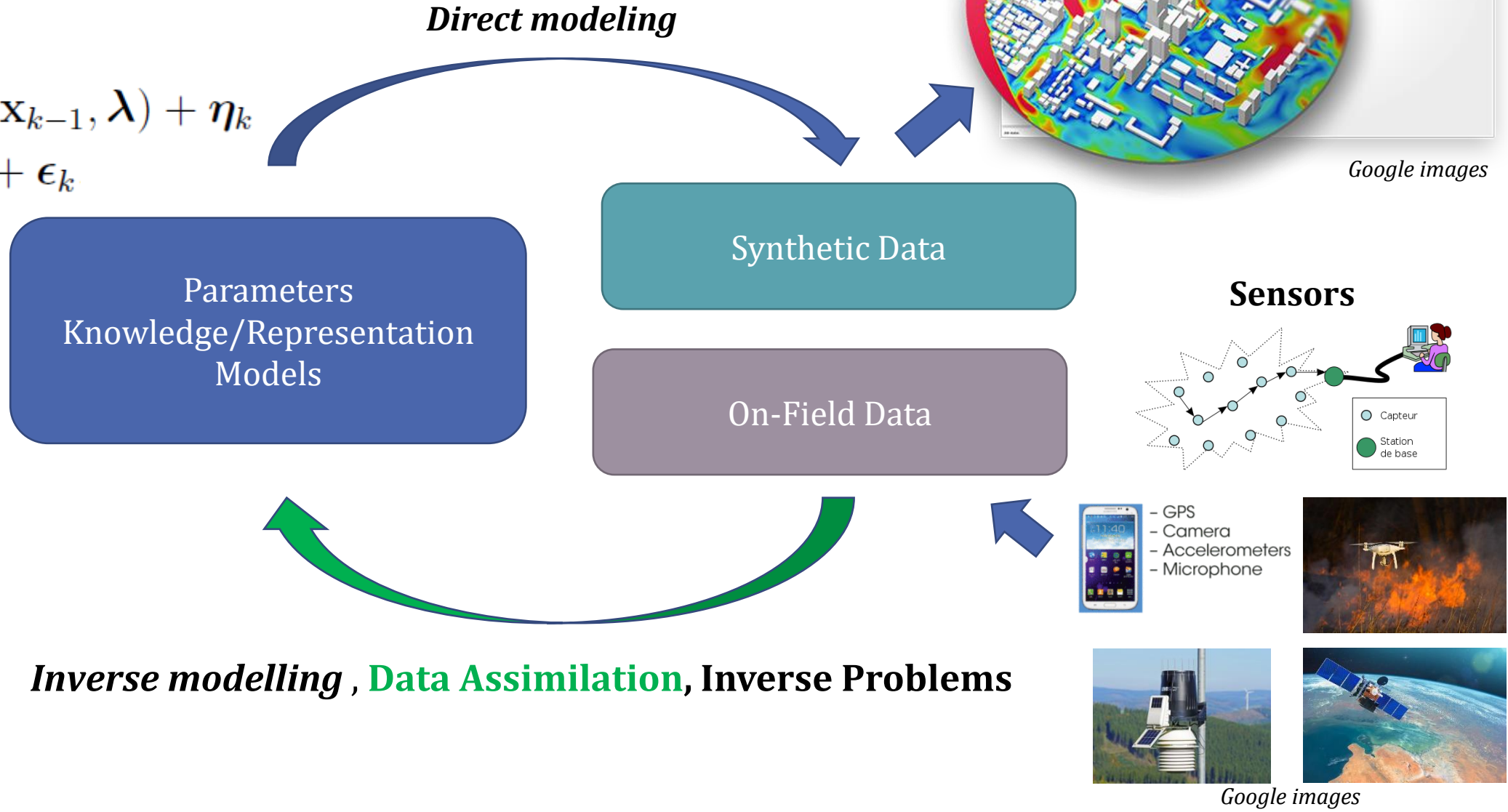
Scientific challenges

Improving the relevance of models
for more realistic scenario building

Quickly extract relevant information from data:
Detect, predict to alert, anticipate

Synergy between models and data

$$\mathbf{x}_k = \mathcal{M}_{k:k-1}(\mathbf{x}_{k-1}, \boldsymbol{\lambda}) + \boldsymbol{\eta}_k$$
$$\mathbf{y}_k = \mathcal{H}_k(\mathbf{x}_k) + \boldsymbol{\epsilon}_k$$



What is **data assimilation**?

Estimate states and/or parameters of a physical system using sensor measurements and a model of the system dynamics

Sensor measurements

An optimization problem

Minimize

$$\mathcal{J}(\mathbf{x}_{K:0}) = \frac{1}{2} \sum_{k=0}^K \|\mathbf{y}_k - \mathcal{H}_k(\mathbf{x}_k)\|_{\mathbf{R}_k^{-1}}^2 + \frac{1}{2} \sum_{k=1}^K \|\mathbf{x}_k - \mathcal{M}_{k:k-1}(\mathbf{x}_{k-1})\|_{\mathbf{Q}_k^{-1}}^2 + \frac{1}{2} \|\mathbf{x}_0 - \mathbf{x}^b\|_{\mathbf{B}^{-1}}^2$$

State at
time K

$$\begin{aligned}\mathbf{x}_k &= \mathcal{M}_{k:k-1}(\mathbf{x}_{k-1}, \boldsymbol{\lambda}) + \boldsymbol{\eta}_k \\ \mathbf{y}_k &= \mathcal{H}_k(\mathbf{x}_k) + \boldsymbol{\epsilon}_k\end{aligned}$$

Dynamical model

Sensor model

Regularization term

✓ Accurate method (with adjoint system)

✓ Significant computation time

What is data for DT and natural hazard monitoring?



<https://www.latimes.com/>

- ✓ Geophysical sensors deployment using Wireless Sensor Networks by satellites
 - ✓ New sources of Data: **Citizen data**
- ## Crowdsensing

Android's earthquake alert system

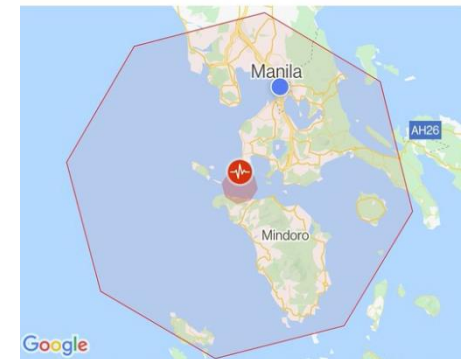


https://www.youtube.com/watch?v=1_9PeG6lcpq

Avoid damaged buildings
Check for cracks and damage. Evacuate if it looks like the building may collapse.

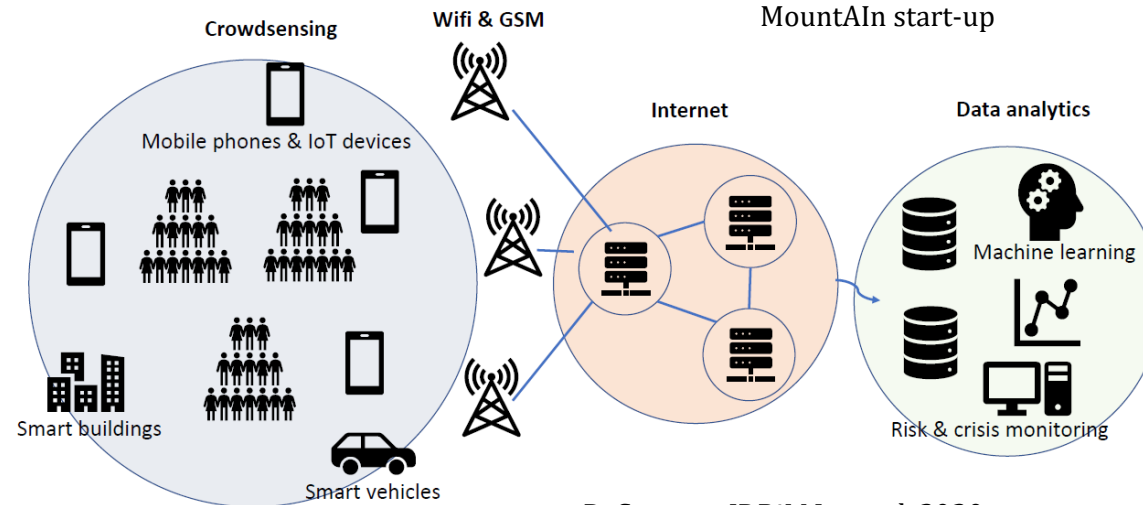
Est. mag 5.5 earthquake

63.5 miles away
July 24, 2021 4:49 AM
Source: Android Earthquake Alerts System



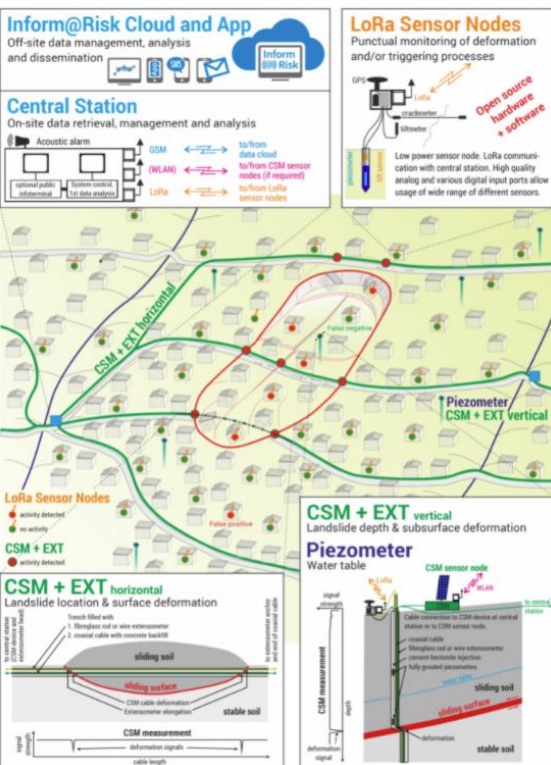
<https://twitter.com>

Internet of Things (IoT)



Sensors : GPS, camera, accelerometer, temperature

D. Georges, IDriM Journal, 2020
<https://doi.org/10.5595/001c.17963>



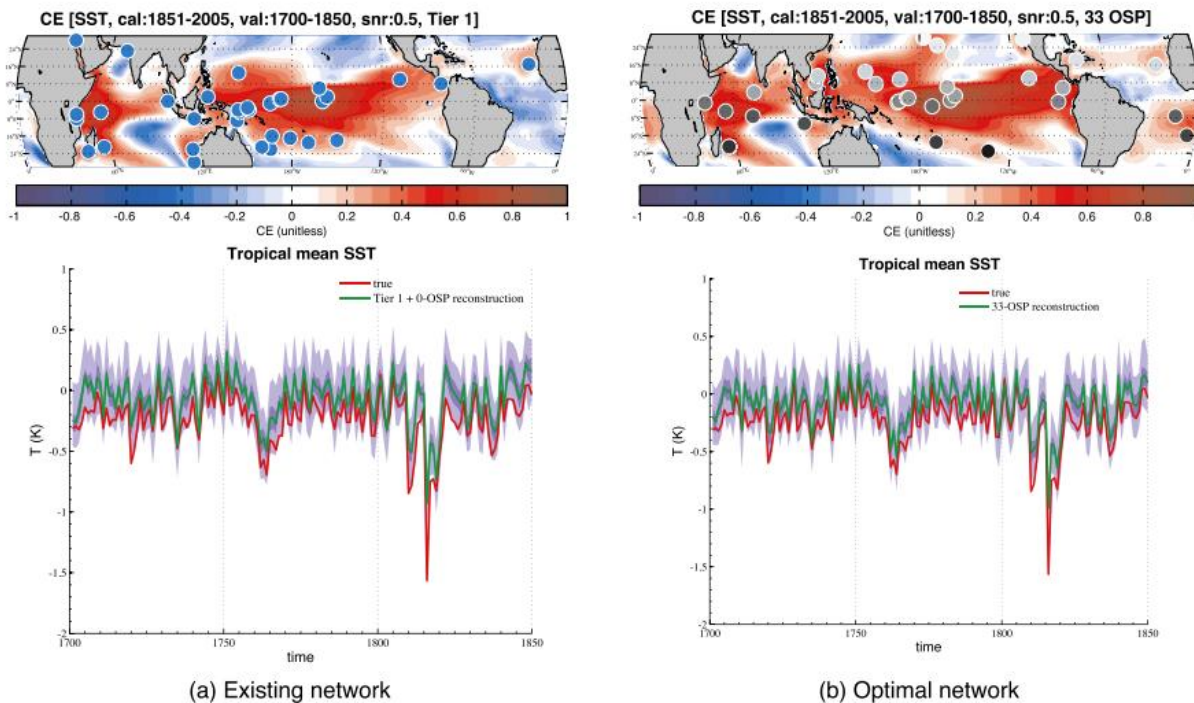
<https://doi.org/10.3390/s21082609>

A challenge: The optimal deployment of sensors

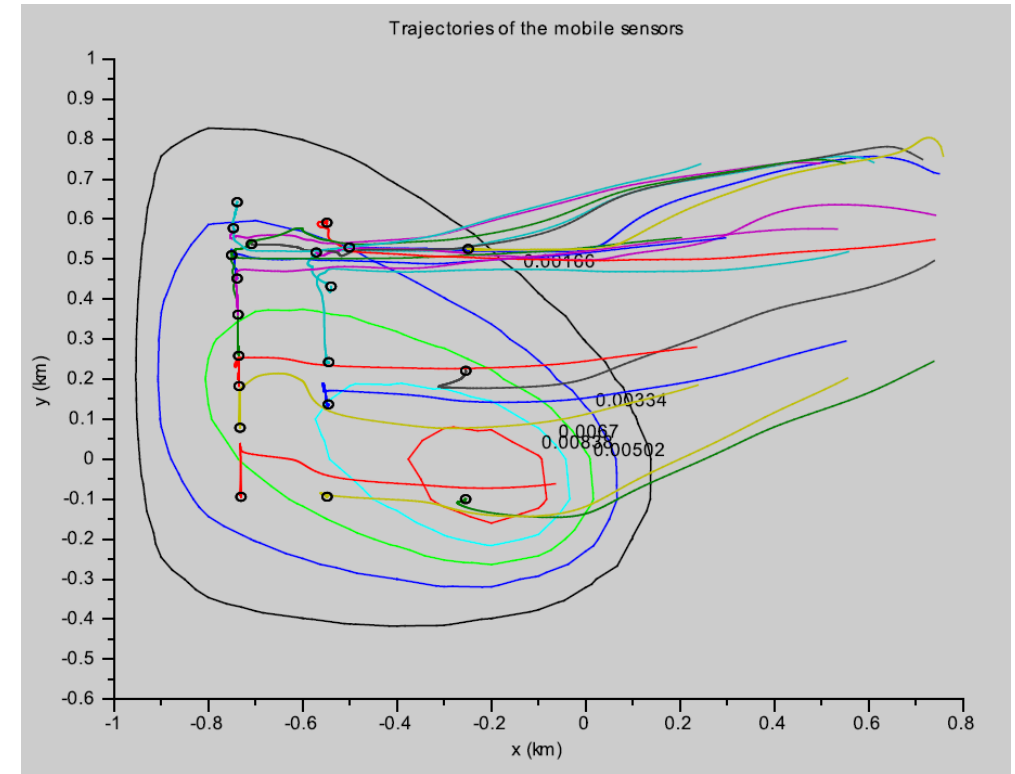
Optimal sensor location

Maximizing an observability indicator: e.g., sensitivity of measurements to parameters/states to be estimated

Optimal network of coral proxies for paleoclimate monitoring



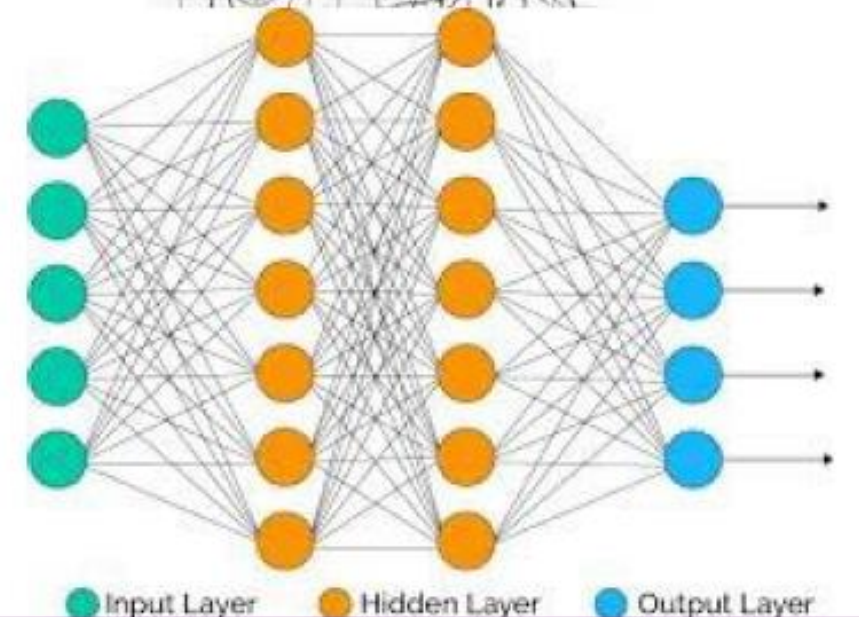
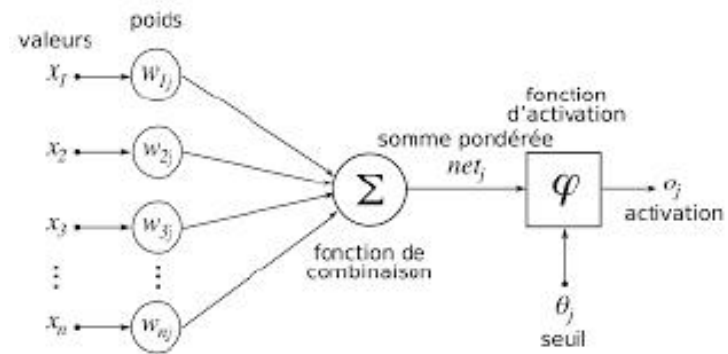
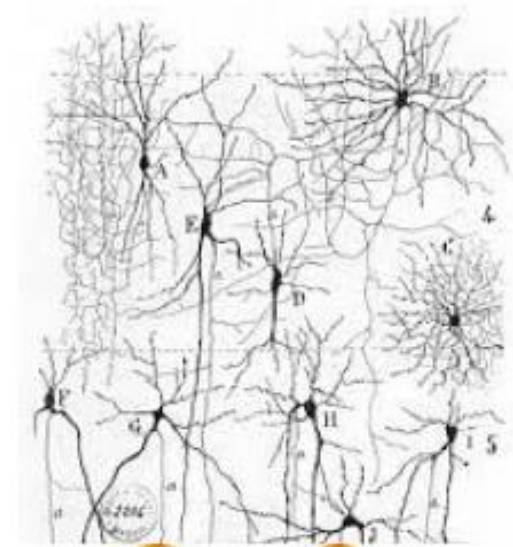
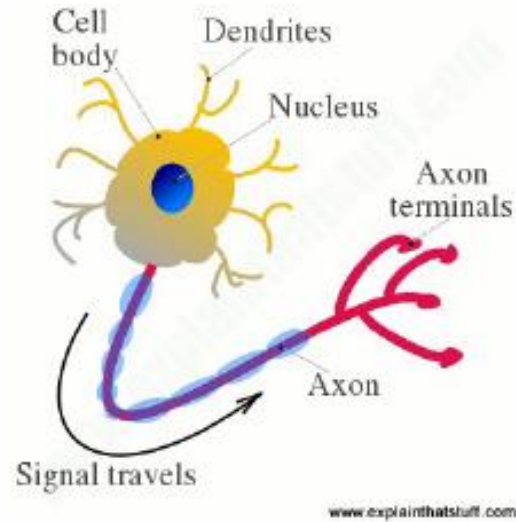
M. Comboul et al, J. of Climate, 2015



Optimal navigation of drones for pollution detection
D. Georges, IDReM Journal, 2020

Deep Learning: What is an **artificial neural network** (ANN)?

An artificial neuron = a mathematical approximation of a biological neuron



Deep Learning: What is **supervised learning** of a ANN?

Finding a relationship between a set of data X and a set of data Y,

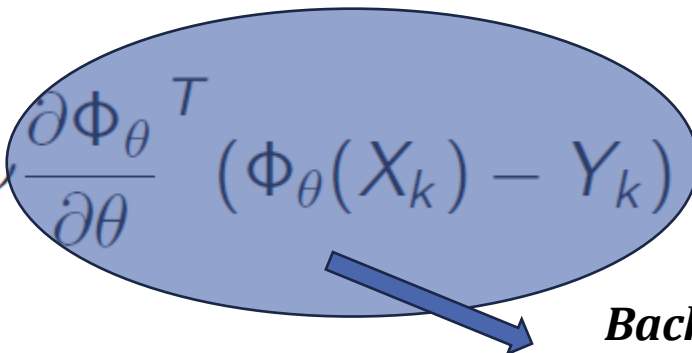
by solving a **nonlinear regression problem**:

Minimization of a loss function

$$\min_{\theta} \frac{1}{2} \sum_{i=1}^L \|\Phi_{\theta}(X_i) - Y_i\|^2$$

Update of the parameters of the neural network,

by using a **stochastic gradient approach**:

$$\theta^{k+1} = \theta^k - \nu \frac{\partial \Phi_{\theta}^T}{\partial \theta} (\Phi_{\theta}(X_k) - Y_k)$$


*Back propagation computation
(Le Cun et al., 1989)*

An example in natural hazard monitoring : Data-based wildfire ignition location

Direct learning of the inverse function based on snapshots of sensor outputs

L. Barbance, S. Benkaddour, S. Drissi, D. Georges, 2019



Google images

y =temperature measurements

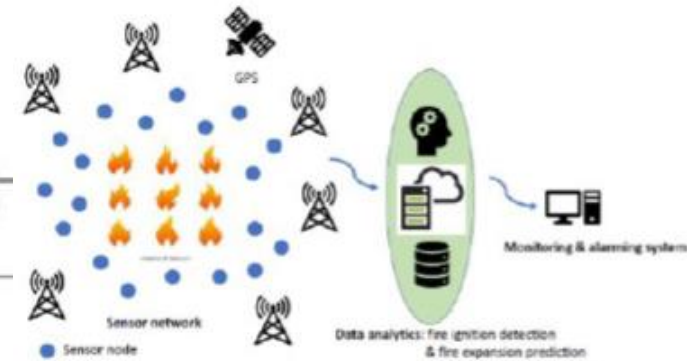
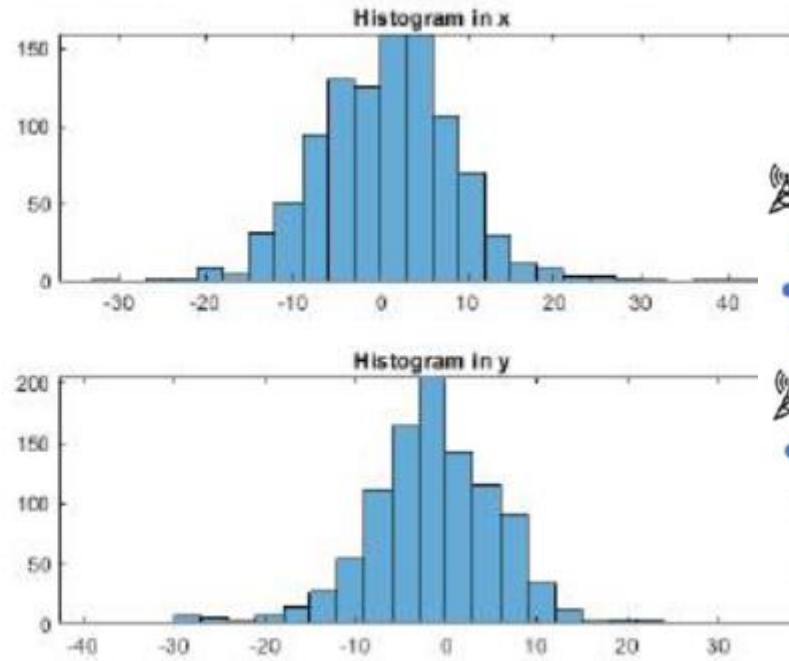
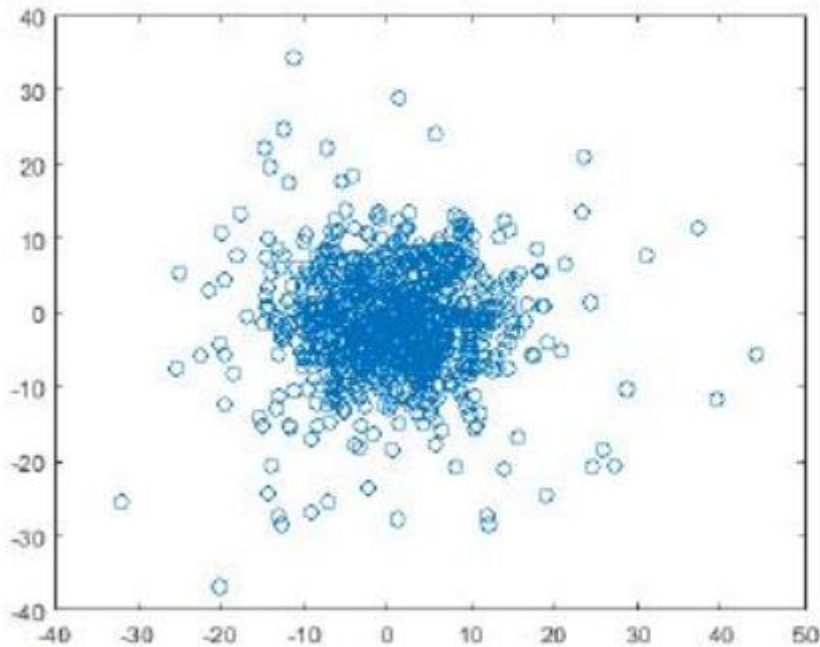
Inverse function

$$p = \mathcal{F}(y)$$

Convolutional
Neural Network

p = fire start location

Distribution of location errors



85 % of location errors < 10 m
With 30 sensors scattered on a cell [500m, 500m]

Learning with synthetic data obtained
from model simulations

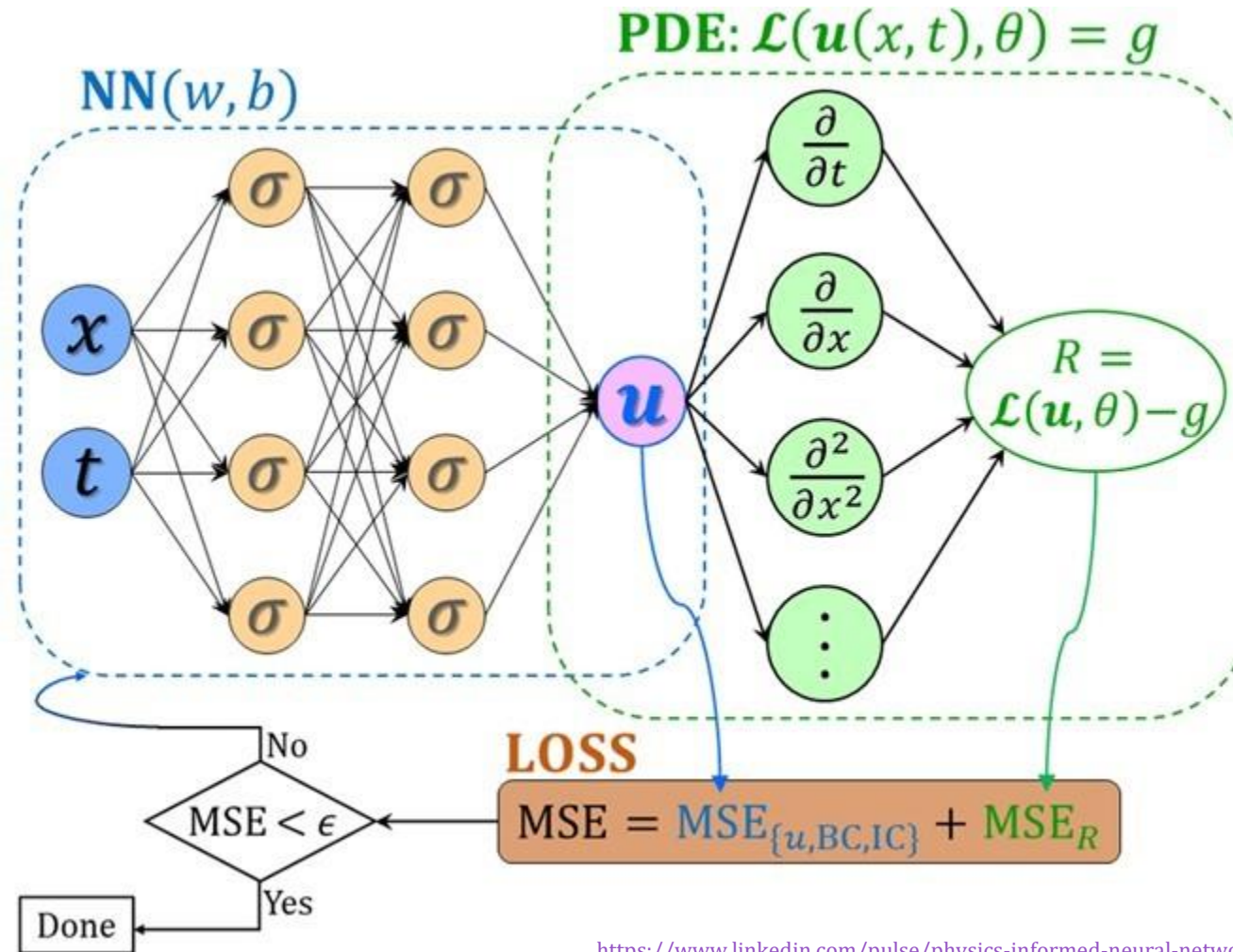
From **data-based** machine learning to **physics-informed** machine learning

- *Bringing together the “best of the two worlds” of modelling:*
- The data is used to “enrich” the solution derived from a deep neural network
- But **the solution complies with the laws of physics**

What is a **Physics-Informed Neural Network (PINN)**?

(Raissi et al., 2019)

The physical model is now included in the loss function



Mathematical formulation of PINNs

If alone, a pure data-based NN learning

$$\min_{(w,b)} MSE_{\{u,BC,IC\}} + MSE_R$$

$$MSE_R = \frac{K_m}{N} \sum_{k=1}^N (\mathcal{L}(u_{NN}(x_k, t_k)), \theta) - g)^2$$

$$MSE_{\{u,BC,IC\}} = \frac{K_d}{M} \sum_{k=1}^M (u_{NN}(x_k, t_k) - y(x_k, t_k))^2 + \frac{K_b}{N} \sum_{k=1}^N (\mathcal{B}(u_{NN}(x_k, t_k)), \theta) - f)^2$$

$y(x_k, t_k)$ is an observable (solution of the PDE obtained from real data and/or simulations).

u_{NN} is the approximation produced by the neural network.

Data assimilation (estimation of the parameters θ and $u(x, t)$ without knowing the initial state $u(x, 0)$) can be solved by considering

$$\min_{(w,b),\theta} MSE_{\{u,BC\}} + MSE_R$$

A direct link with the classical **data assimilation** approach

Similar to the first term of $\text{MSE}_{\{u, BC\}}$

Similar to MSE_R

Minimize

$$\mathcal{J}(\mathbf{x}_{K:0}) = \frac{1}{2} \sum_{k=0}^K \|\mathbf{y}_k - \mathcal{H}_k(\mathbf{x}_k)\|_{\mathbf{R}_k^{-1}}^2 + \frac{1}{2} \sum_{k=1}^K \|\mathbf{x}_k - \mathcal{M}_{k:k-1}(\mathbf{x}_{k-1})\|_{\mathbf{Q}_k^{-1}}^2 + \frac{1}{2} \|\mathbf{x}_0 - \mathbf{x}^b\|_{\mathbf{B}^{-1}}^2$$

State at
time K

$$\begin{aligned}\mathbf{x}_k &= \mathcal{M}_{k:k-1}(\mathbf{x}_{k-1}, \boldsymbol{\lambda}) + \boldsymbol{\eta}_k \\ \mathbf{y}_k &= \mathcal{H}_k(\mathbf{x}_k) + \boldsymbol{\epsilon}_k\end{aligned}$$

Dynamical model

Sensor model

A case study – DT for flood monitoring based on a 1D runoff model

A simplification of shallow water equations:

The kinematic wave equation

Physical model

$$\alpha\beta Q(x,t)^{\beta-1} \frac{\partial Q}{\partial t}(x,t) + \frac{\partial Q}{\partial x}(x,t) = 0, x \in [0, L], t \in (0, T),$$
$$Q(x=0, t) = f(t) \text{ (BC)},$$
$$Q(x=L, t) = Q_0(x) \text{ (IC)}$$

$Q(x, t)$ is the water flow rate, as a function of space coordinate x and time t .

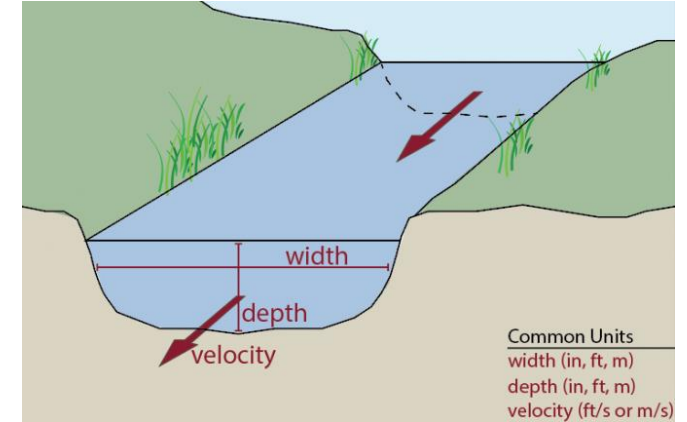
$$\beta = 0.6 \text{ and } \alpha = \left(\frac{nb^{2/3}}{\sqrt{S_0}} \right)^{3/5}$$

n is a roughness coefficient, b is the width of the channel, S_0 is the slope of the channel,

$f(t)$ is the upstream flow rate defining the boundary condition, and $Q_0(x)$ is the initial flow rate distribution.



Guadalupe flash flood – Texas, July 2025 – abc news



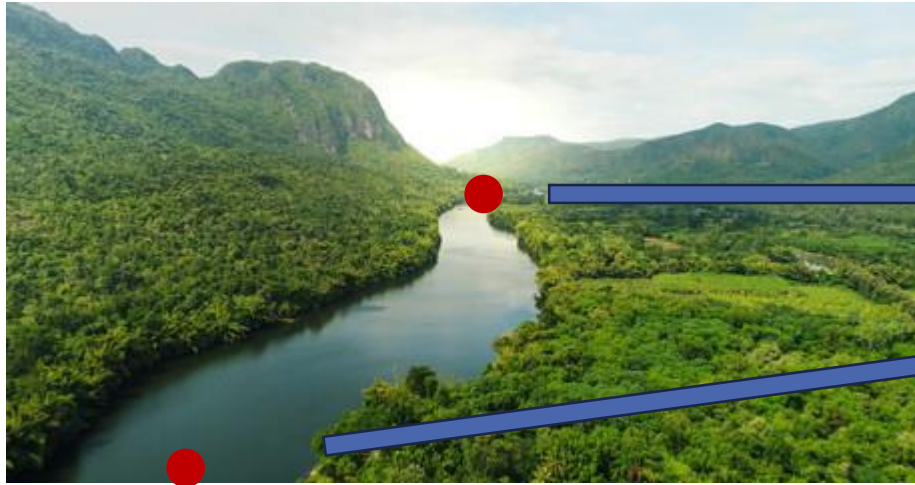
Google images

Data : Flow rate

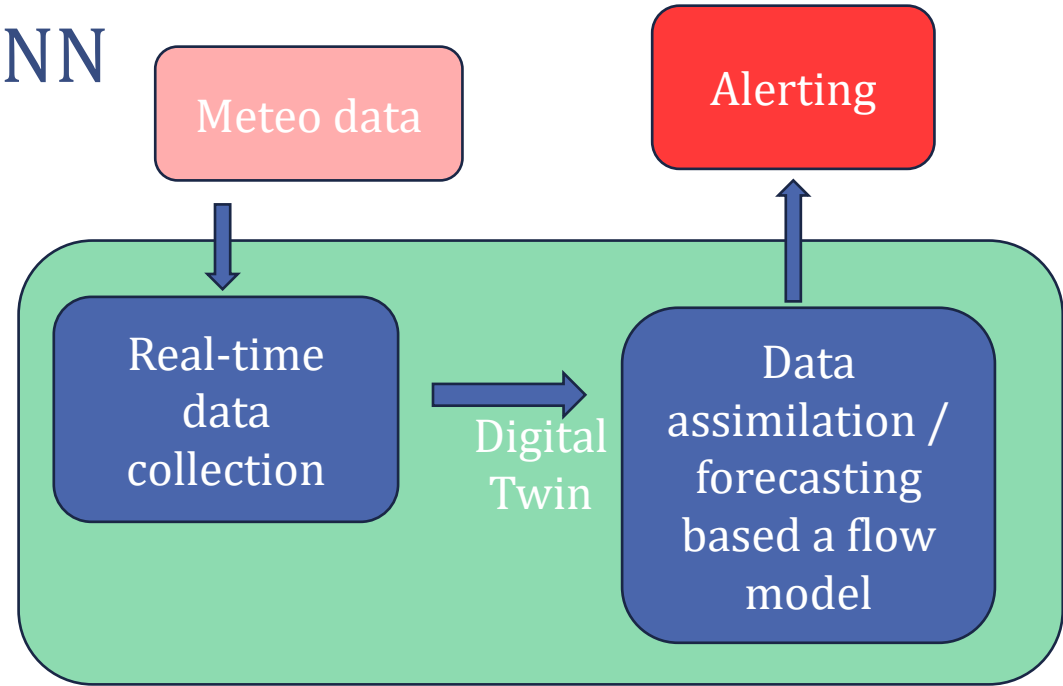
$$y_i^Q(t) = Q(x = x_i^s, t)$$

x_i^s is the location of the sensor i

DT: Flood monitoring using a PINN

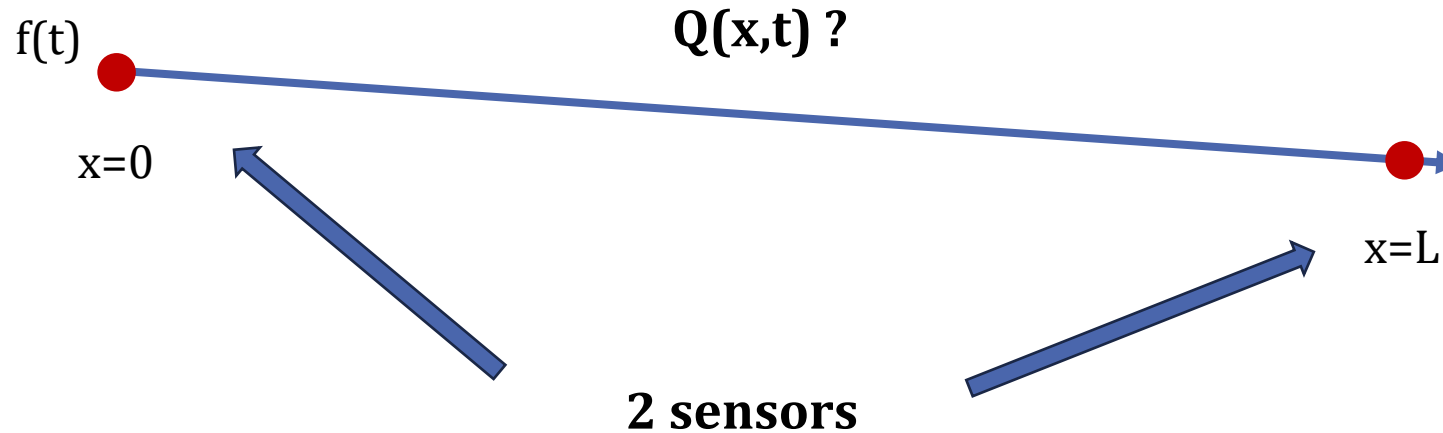


Google images



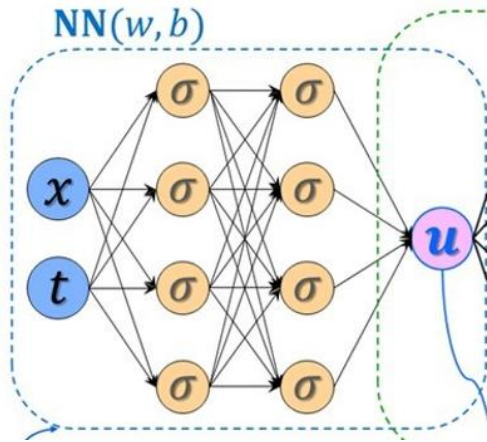
Receding horizon approach

From only 2 sensors at the extremities,
can we estimate the flow rate $Q(x,t)$ for all x and t ?



PINN formulation for flood monitoring

<https://www.linkedin.com/pulse/physics-informed-neural-network-pinn-une-r%C3%A9volution-la-mitchozounou-0eaje/>



$$\min_{(w,b)} MSE_{\{u,BC\}} + K_r \times MSE_R$$

$$MSE_R = \frac{1}{N} \sum_{k=1}^N (\alpha \beta Q_{NN}(x_k, t_k)^{\beta-1} \frac{\partial Q_{NN}}{\partial t}(x_k, t_k) + \frac{\partial Q_{NN}}{\partial x}(x_k, t_k))^2$$

PDE residual

$$MSE_{\{u,BC\}} = \frac{1}{M} \sum_{i=1}^{N_s} \sum_{k=1}^M (Q_{NN}(x_i^s, t_k) - y_i^Q(t_k))^2 + \frac{1}{M} \sum_{k=1}^M (u_{NN}(0, t_k) - f(t_k))^2$$

Data sensor fitting term

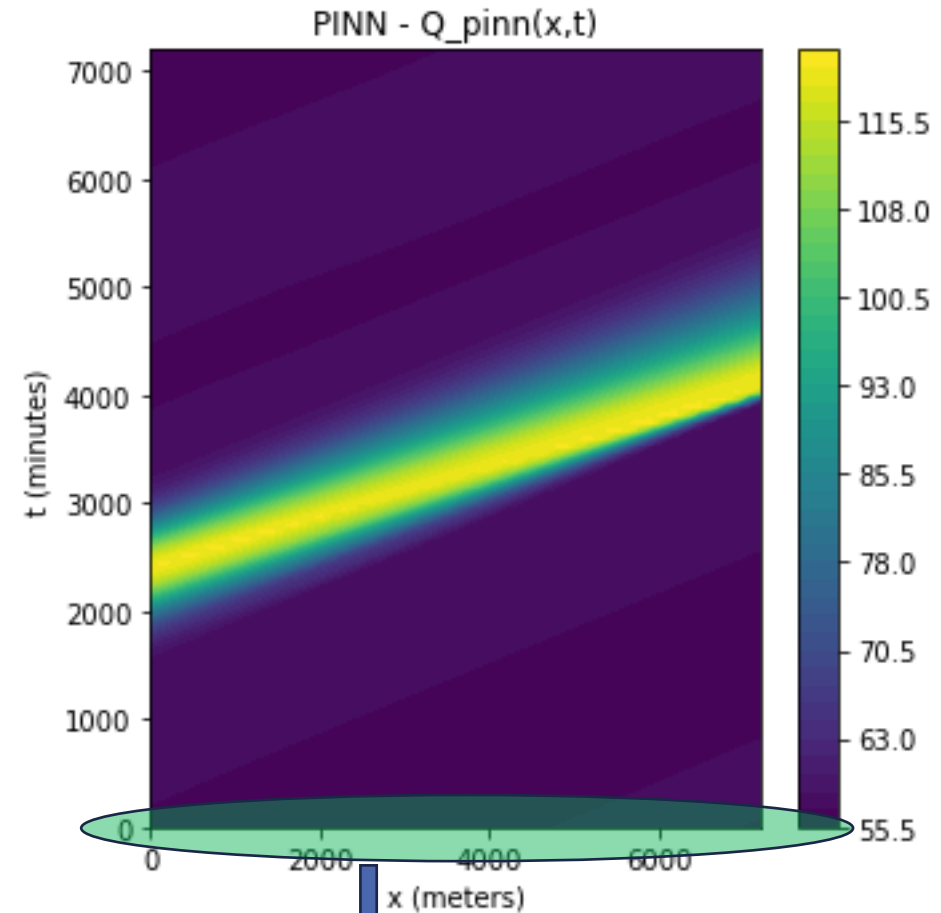
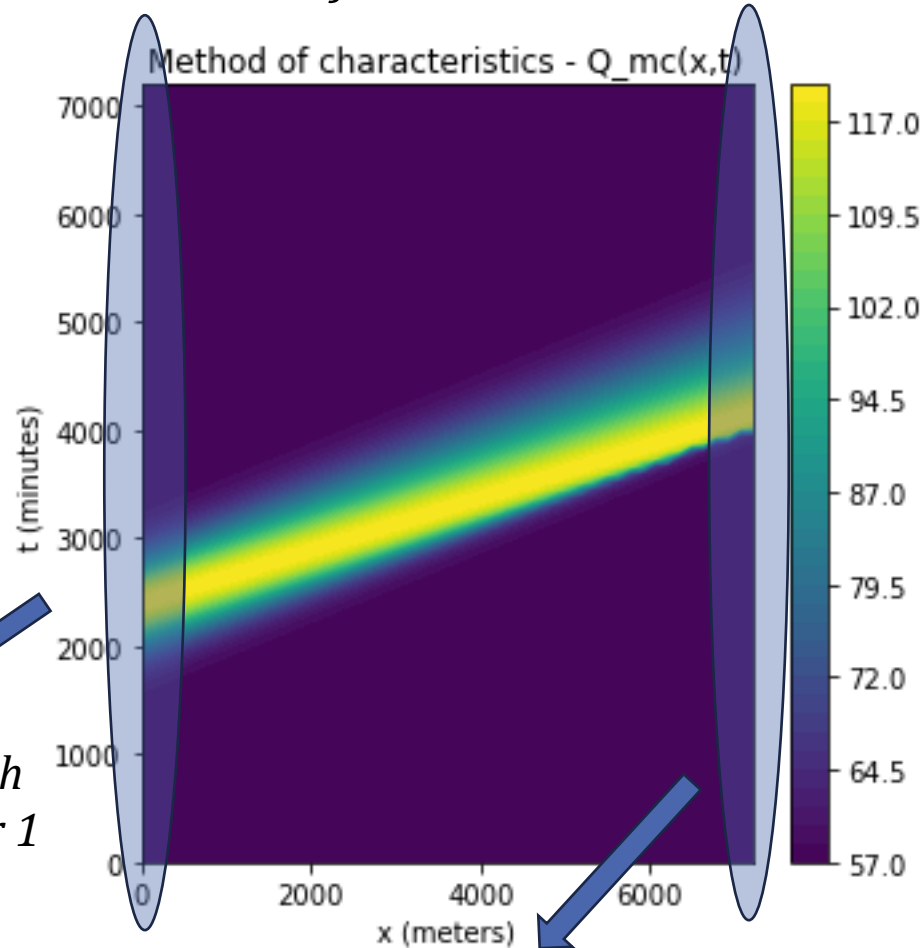
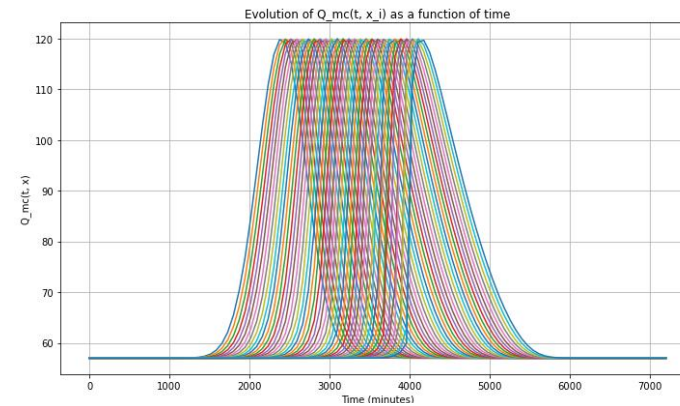
BC residual

NN: 1 hidden layer with 100 neurons

Results

Channel of 7200m with width of 60m,
Gaussian distribution of the inflow hydrograph

- The « real » data is obtained from a simulation based on the *method of characteristics*



Good estimation of the unknown initial state!

Conclusions

- **Machine learning is set to play an important role in the development of digital twins:**

The synergy between models and data is facilitated by PINNs:

Multi resolution, no need for adjoint system calculation, meshless approach, complexity independent of dimensions...

- **Many challenges remain, amongst them:**

Efficient learning of parametrized solutions for complex physical models:

high-dimensional simulation/estimation problem for a variety of data and parameters,

very heterogeneous domains (many internal BCs),

improvement through optimal sensor location ...

Thank you for your attention!

